Faculty of Natural Sciences

IMPORTANT NOTES

If for one subject you can find several different types (lecture, practice, laboratory) of courses then please choose one and only one course from each type in order to be able to perform the subject's requirements successfully.Civil Engineering courses are on the website seperately. Courses chosen from the offer of Faculty of Civil Engineering will be checked and arranged individually by the departmental coordinator.

Subject code	Subject name	Requirement	ECTS credit
BMETE11AF11	Applied Solid State Physics	Exam	2

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	THU:10:15-12:00(F3213)

Band structure of metals and semiconductors, electron transport, electron scattering mechanisms, 2 dimensional electron gases, Si technology (FET, SSD memory), semiconductor heterostructure (semiconductor laser, MEMT), nanoelectronics, single electron transistor. – Magnetic materials, origin of magnetic momentum and interaction between moments, magnetic structures. Magnetism of metals, spin polarized bands, spintronics devices (spin valve, MRAM). Spin transistor, magnetic semiconductors.
– Jenő Sólyom: Fundamentals of the Physics of Solids (Springer 2007) – Thomas Ihn: Semiconductor Nanostructures: Quantum States and Electronic (2009)

Subject code	Subject name	Requirement	ECTS credit
BMETE11AP60	Thermodynamics and Statistical Physics	Exam	6

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	THU:10:15-12:00(F29)
Practice	T2	English	THU:08:15-10:00(KF84)
Practice	T1	English	THU:08:15-10:00(F3213)

Introductory course to experimental physics, with special emphasis on the physical phenomena and demonstrations. Temperature and the Zeroth Law of Thermodynamics. Temperature scales. The concept and description of the ideal gas. Thermodynamic state and processes. Heat, specific heat, latent heat, internal energy, and the First Law of Thermodynamics. Generalized work. Processes with ideal gas. The kinetic theory of gases, description of pressure, temperature, internal energy, and molar specific heat of an ideal gas. Equipartition of Energy. Real gases and the van der Waals gas. The barometric formula and the Boltzmann distribution. The Maxwell distribution of molecular speeds and its measurement. The mean free path approximation, diffusion, heat transfer, and viscosity. Closed cycles with ideal gas and the Carnot cycle. Heat engines and heat pumps. The Second Law of Thermodynamics, the Carnot principle, reversible, and irreversible processes. The thermodynamic temperature scale. Examples of heat engines. The Clausius inequalities, the entropy, principle of entropy growth. The fundamental equation of thermodynamics, thermodynamic potentials, their differential relations, and the Maxwell relations. The Third Law of Thermodynamics and its consequences. Phase changes in gases, the Clausius–Clapeyron equation. Principles of statistical physics: micro- and macro-states, the statistical description of entropy, quantum statistics.
– Raymond A. Serway, John W. Jewett: Physics for Scientists and Engineers (Cengage Learning; 10th edition, 2018) ISBN 978-1337553278 Ch 19-22
– Raymond A. Serway , Clement J. Moses, Curt A. Moyer: Modern Physics (Thomson Learning, 2005, 3rd Edition), ISBN 0-534-49339-4
– Herman Gewirtz, Jonathan S. Wolf: Barron's SAT Subject Test in Physics 9th Edition (Barron’s, 2010) ISBN 978-0-7641-4353-3 ezt nem tudom, ezt egy tesztkönyv?

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF12	Group Theory in Solid State Research	Exam	3

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	THU:14:15-16:00

<span lang="EN-US" style="font-size:11.0pt;font-family: "Times New Roman","serif";mso-fareast-font-family:"Times New Roman";mso-ansi-language: EN-US;mso-fareast-language:HU;mso-bidi-language:AR-SA">Introduction: point groups, fundamental theorems on finite groups, representations, character tables. Optical spectroscopy: selection rules, direct product representations, factor group. Electronic transitions: crystal field theory, SO(3) and SU(2) groups, correlation diagrams, crystal double groups. Symmetry of crystals: space groups, International Tables of Crystallography. Electronic states in solids: representations of space groups, compatibility rules.

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF24	Transport in Complex Nanostructures	Exam	3

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	THU:09:15-11:00(KF85)

The course overviews the complex physical phenomena in various hybrid nanostructures with a special emphasis on the following topics of superconductiong nanostructures and spintronics:
Introduction to mesoscopic superconductivity. Andreev reflections, BTK theory and mesoscopic proximity effects. Multiple Andreev Reflections. Advanced applications of the Josephson effect. Investigation of Andreev Bound states and the current-phase relation. Andreev Qubits. Superconducting islands, Andreev states in quantum dots. Majorana fermions.
Basic concepts of spintronics. Magnetization measurements: magnetic force microscopy, scanning NV center methods, X-ray magnetic circular dichroism, etc. Magnetoresistance phenomena (AMR, GMR, TMR). Spin injection, non-local measurements. Semiconductor spintronics, Rashba effect, spin relaxation, weak anti-localization. Spintronics in quantum dots. Optical spin injection, electron spin resonance. Spin Hall phenomena. Exotic spin structures, multi ferroic materials, skyrmions. Antiferromagnetic spintronics. Spin transfer torque, spin pumping.

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF52	Modern Semiconducting Devices	Exam	3

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	WED:08:15-10:00(FASEM)

<span style="font-size:11.0pt;font-family:"Times New Roman","serif"; mso-fareast-font-family:"Times New Roman";mso-ansi-language:HU;mso-fareast-language: HU;mso-bidi-language:AR-SA">The course introduces the hardware building blocks of modern information technologies from traditional semiconductor architectures to the the most up-to-date concepts, technologies and devices. Topics: History of semiconductor devices and semiconductor industry. Advanced silicon technologies from crystal growth to micromachining and nanofabrication techniques. Si devices from traditional MOS FETs to trigate transistors or CCD sensors. Memory devices (SRAM, DRAM, flash). Si solar cells. Compound semiconductors, band engineering, two dimensional electron gas systems, quantum wells, light emitting and laser diodes, high electron mobility transistors, GaN technology. Organic semiconductors: polymer solar cells, OLEDs, printed electronics. Perovskite solar cells. Sensors and actuators: MEMS, physical, chemical, biological sensors, actuators, robotic applications, biointerfaces, artificial skin and nose. Novel device platforms: spintronic devices and resistive switching memories. Novel computing architectures: brain inspired computing, in memory computing, hardware implementation of artificial neural networks.

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF53	Fundamentals of Nanophysics	Exam	5

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	WED:10:15-13:00(F3M01)

<span style="font-size:11.0pt;font-family:"Times New Roman","serif"; mso-fareast-font-family:"Times New Roman";mso-ansi-language:HU;mso-fareast-language: HU;mso-bidi-language:AR-SA">The building blocks of nowadays electronic devices have already reached a few tens on nanometers sizes, and further miniaturization requires the introduction of novel technologies. At such small length-scales the coherent behavior and the interaction of electrons, together with the atomic granularity of matter induce several striking phenomena, that are not observed at the macroscopic scale. The course gives an introduction to a broad set of nanoscale phenomena covering the following topics: characteristic length-scales; basic concepts of quantum transport, conductance quantization; coherent and incoherent transport, interference phenomena in nanostructures; mesoscopic phenomena in atomic and molecular nanojunctions; quantized Hall effect; noise phenomena in nanostructures; graphene nanostructures, 2D heterostructures; quantum dots.

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF54	Optical Spectroscopy in Materials Science	Exam	5

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	FRI:09:15-12:00

<span lang="EN-US" style="font-size:11.0pt;font-family: "Times New Roman","serif";mso-fareast-font-family:"Times New Roman";mso-ansi-language: EN-US;mso-fareast-language:HU;mso-bidi-language:AR-SA">Electromagnetic waves in vacuum and in a medium; complex dielectric function, interfaces, reflection and transmission. Optical conduction in dipole approximation; linear response theory, Kramers-Kronig relation, sum rules. Simple optical models of metals and insulators; Drude model, Lorentz oscillator. Optical phonons, electron-phonon interaction. Optical spectroscopes: monochromatic- and Fourier transformation spectrometers. Optical spectroscopy of interacting electron systems: excitons, metal-insulator transition, superconductors. Magneto optics: methods and current applications.

Subject code	Subject name	Requirement	ECTS credit
BMETE11MF57	Theory of Magnetism	Exam	5

Course type	Course code	Course language	Timetable information
Lecture	T0	English	THU:11:15-13:00(F3M01)
Practice	T1	English	THU:13:15-14:00(F3M01)

<span lang="EN-US" style="font-size:11.0pt;font-family: "Times New Roman","serif";mso-fareast-font-family:"Times New Roman";mso-ansi-language: EN-US;mso-fareast-language:HU;mso-bidi-language:AR-SA">Magnetic phenomena are considered as electron correlation effects. This course builds heavily on knowledge gained by successful completion of the course “Modern solid state physics”. The following topics are discussed: Landau levels in magnetic field, magnetism of extended electron states, magnetism of atoms and ions, magnetite, direct exchange, kinetic exchange, Mott transition, Mott insulators, mean field theory of magnetic ordering, the ferromagnetic Heisenberg model, the antiferromegnetic Heisenberg model.

Subject code	Subject name	Requirement	ECTS credit
BMETE14MX00	Modern Physics for Chemical Engineers	Exam	3

Course type	Course code	Course language	Timetable information
Lecture	E0	English	MON:12:15-14:00; THU:10:15-12:00

Topics:The course covers introductions to two disciplines: Quantum Mechanics and Solid State Physics. After the semester students should be able to understand the basic principles behind these two disciplines and solve some simple quantum mechanical and solid state physics problems. This will contribute to the understanding of the workings of modern electronics and nanotechnology. To follow the course no higher mathematics than algebra and the basics of the differential and integral calculus is required.

Detailed thematics:

Quantum Mechanics. Blackbody radiation, photoelectric effect, Compton effect, stability and line spectra of atoms, Frank-Hertz experiment, Time dependent and independent Schrödinger's equation, stationary states, wave function, "wave - particle duality", electron diffraction, two-slit experiment, uncertainty relations, electron wavefunction probability distribution in an atom, solving the Schrödinger equation, tunneling, the ammonia molecule, electron emission from metals, perturbation calculus, selection rules, operator calculus, eigenstate problems, measurement, quantum mechanics of the hydrogen atom, quantum numbers, H spectrum and selection rules, electron spin, Zeeman-effect, Stern-Gerlach experiment, spin-orbit coupling, atoms with more than one electron, the exclusion principle, indistinguishable particles, periodic table of elements, buildup of shells, Hund's rule, valence and core electrons, molecules, molecular orbitals, chemical bonding, H-H bond, H<span lang="EN-US" style="font-size:9.5pt;mso-bidi-font-size:12.0pt;line-height:115%; position:relative;top:1.0pt;mso-text-raise:-1.0pt">2+ molecule ion, bonding and anti-bonding states, orbital hybridisation, heteronuclear molecules, sp3 hybridization, rotation and vibration of molecules, Franck-Condon principle, Rayleigh and Raman scattering, Stokes and anti-Stokes scattering, Statistical physics. Classical and quantum statistics. Distribution functions, distinguishable and indistinguishable particles, photon gas, Einstein model, laser principle. Solid State Physics. Short and long range ordering, amorphous and crystalline solids, crystalstructures, lattices (point lattice and basis), symmetries and unit cells,primitive, conventional and Wigner-Seitz cells, primitive vectors, Millerindexes, Bravais lattices, close packing structures, reciprocal lattice,k-space, X-ray diffraction, Laue formulae, classical physical models forcrystals: lattice vibrations, monatomic and diatomic linear chain model,boundary conditions, form of the solution, dispersion relation, generalizationfor 3 dim., QM handling of lattice vibrations, phonons, momentum and energy ofphonons, relative to the momentum and energy of Bloch electrons, specific heatof solids, equipartition principle and the Debye model, specific heat fromelectrons, conductors and insulators, band theory of solids, formation ofbands, insulators, conductors, real band structures, conduction models,Drude model, collision time, mean free path, Wiedmann-Franz law, Sommerfeldmodel of metals, Fermi energy, electrons and holes, equivalence of electron andhole conductivity in a completely filled band, metals with hole conduction,work function, thermionic emission, contact potential, crystal potential,double layer at the surface, Bloch functions, Hartree-Fock method, dispersionrelation, Brillouin zone, reduced zone picture, kinematics of electrons andholes, Bloch oscillations, effective mass, tight binding model, semiconductors,intrinsic conductivity, density of states in the conduction and valence bands,position of the Fermi level, donors and acceptors, charge carrierconcentrations, extrinsic conductivity, Fermi level in doped semiconductors,p-n junction, application of p-n junctions, diode, (MOS)FET, bipolartransistors, Schottky and ohmic structures, characteristics.

Subject code	Subject name	Requirement	ECTS credit
BMETE15MF10	Random Matrix Theory and Its Physical Applications	Exam	3

Course type	Course code	Course language	Timetable information
Lecture	T0	English	TUE:08:15-10:00

Random matrix theory provides an insight of how one can achieve information relatively simply about systems having very complex behavior. The subject based on the knowledge acquired in quantum mechanics and statistical physics together with some knowledge of probability theory provides an overview of random matrix theory. The Dyson ensembles are defined with their numerous characteristics, e.g. the spacing distribution, the two-level correlation function and other quantities derived thereof. Then the thermodynamic model of levels is obtained together with several models of transition problems using level dynamics. Among the physical applications the universality classes are identified in relation to classically integrable and chaotic systems. The problem of decoherence is studied as well. Then the universal conductance fluctuations in quasi-onedimensional disordered conductors are investigated. Other models are investigated: the disorder driven Anderson transition and the random interaction model of quantum dot conductance in the Coulomb-blockade regime. We use random matrix models to investigate chirality in two-dimensional and Dirac systems and the normal-superconductor interface. The remaining time we cover problems that do not belong to strictly physical systems: EEG signal analysis, covariance in the stock share prize fluctuations, mass transport fluctuations, etc.

Subject code	Subject name	Requirement	ECTS credit
BMETE15MF11	Evolutionary Game Theory	Exam	3

Course type	Course code	Course language	Timetable information
Lecture	T0	English	FRI:10:15-12:00

The main goal of this course is to demonstrate the ways how the game theory and evolutionary game theory describe real-life situations affecting human behavior, economics, and biological systems. After a brief survey of the basic concept of the traditional game theory (e.g., games, strategies, Nash equilibrium, etc.) we will study evolutionary games that combine the concepts of game theory with the spirit of Darwinism. We will discuss the decomposition of games and also the potential games related to physical systems. Using simple multi-agent mathematical models we will investigate the effects supporting the maintenance of cooperative behavior in the situations of different social dilemmas (e.g., prisoner's dilemma or public goods game) when the individual interests prefer defection to cooperation. The predictions of the mathematical models will be contrasted with human and animal experiments. Finally we study systems where the evolution is controlled by the competition between different spatial strategy associations.

Subject code	Subject name	Requirement	ECTS credit
BMETE15MF74	Computer Simulation in Physics	Mid-semester mark	5

Course type	Course code	Course language	Timetable information
Lecture	TA0	English	THU:16:15-18:00(F3213)
Practice	TA1	English	THU:18:15-19:00(F3213)

Subject code	Subject name	Requirement	ECTS credit
BMETE80AF45	Monte Carlo Methods	Exam	4

Course type	Course code	Course language	Timetable information
Practice	T1	English	WED:08:15-10:00(R214)

Subject code	Subject name	Requirement	ECTS credit
BMETE80BE04	Monte Carlo Methods	Mid-semester mark	4

Course type	Course code	Course language	Timetable information
Practice	T1	English	WED:08:15-10:00(R214)

Subject code	Subject name	Requirement	ECTS credit
BMETE80MFAD	Monte Carlo Methods	Mid-semester mark	5

Course type	Course code	Course language	Timetable information
Laboratory	T1	English	WED:08:15-10:00(R214)
Lecture	T0	English	WED:12:15-14:00(R214)

Subject code	Subject name	Requirement	ECTS credit
BMETE80MFAR	Nuclear Medicine	Exam	4

Course type	Course code	Course language	Timetable information
Laboratory	T1	English	MON:16:15-17:00(R438)
Lecture	T0	English	MON:14:15-16:00(R438)

Subject code	Subject name	Requirement	ECTS credit
BMETE80MX07	Radiation Protection	Exam	3

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	T0	English	FRI:10:15-12:00(R214)

Physical fundamentals of generating ionizing radiations: radioactivity, radioactive decay, operation of equipment for generating ionizing radiations. Definition of doses. Biological effects of ionizing radiations: deterministic and stochastic effects, somatic and genetic effects. Control of applications of ionizing radiations in connection with the explanation of generic principles of radiation protection (justification, optimization, and individual limitations). Procedures and conditions of generating ionizing radiations: external and internal exposure situations, natural and artificial radioactivity. Practical implementation of radiation protection: workplace and environmental radiation protection, monitoring, management and disposal of radioactive wastes, applications of radiation shielding. management of nuclear and radiological emergencies.
H. Cember, T.E. Johnson: Introduction to Health Physics

Subject code	Subject name	Requirement	ECTS credit
BMETE90AX02	Mathematics A2a - Vector Functions	Exam	6

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	EN0-EMK	English	MON:16:15-18:00(K375); MON:16:15-18:00(K375); THU:16:15-18:00(K375); THU:16:15-18:00(K375)
Practice	EN1-EMK	English	WED:16:15-18:00(K371); WED:16:15-18:00(K371)

Solving systems of linear equations: elementary row operations, Gauss-Jordan- and Gaussian elimination. Homogeneous systems of linear equations. Arithmetic and rank of matrices. Determinant: geometric interpretation, expansion of determinants. Cramer's rule, interpolation, Vandermonde determinant. Linear space, subspace, generating system, basis, orthogonal and orthonormal basis. Linear maps, linear transformations and their matrices. Kernel, image, dimension theorem. Linear transformations and systems of linear equations. Eigenvalues, eigenvectors, similarity, diagonalizability. Infinite series: convergence, divergence, absolute convergence. Sewuences and series of functions, convergence criteria, power series, Taylor series. Fourier series: axpansion, odd and even functions. Functions in several variables: continuity, differential and integral calculus, partial derivatives, Young's theorem. Local and global maxima / minima. Vector-vector functions, their derivatives, Jacobi matrix. Integrals: area and volume integrals.

Subject code	Subject name	Requirement	ECTS credit
BMETE90AX17	Mathematics A2c	Exam	6

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	EN0	English	
Practice	EN1	English	

Differential calculus of functions of several variables: partial derivatives, differentiability, tangent plane. Derivatives of composite functions. Local and global maxima / minima. Inverse function, implicit function. Double and triple integrals. (5 weeks)
Numerical series, power series, Taylor series. (2 weeks)
Laplace and Fourier transform. (1 week)
Linear algebra. Vectors, applications in geometry. Systems oflinear equations. (3 weeks).
Differential equations (separable differential equations, first order linear differential equations, second order linear differential equations with constant coefficients). (3 weeks)

Subject code	Subject name	Requirement	ECTS credit
BMETE90AX34	Mathematics EP2	Mid-semester mark	2

Course type	**Course code**	**Course language**	**Timetable information**
Practice	EN1	English	WED:08:15-10:00(K221)

Limit, continuity, partial derivatives and differentiability of functions of multiple variables. Equation of the tangent plane. Local extrema of functions of two variables. Gradient and directional derivative. Divergence, rotation. Double and triple integrals and their applications. Polar coordinates. Substitution theorem for double integrals. Curves in the 3D space, tangent line, arc length. Line integral. 3D surfaces. Separable differential equations, first order linear differential equations. Algebraic form of complex numbers. Second order linear differential equations with constant coefficients. Taylor polynomial of exp(x), sin(x), cos(x). Eigenvalues and eigenvectors of matrices.

Subject code	Subject name	Requirement	ECTS credit
BMETE90AX57	Calculus 2 for Informaticians	Exam	6

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	EN0	English	MON:12:15-14:00(E1C); TUE:10:15-12:00(E1C)

Practice	EN1	English	THU:14:15-16:00(IB141)
Practice	EN2	English	THU:14:15-16:00(IB142)

Differential equations: Separable d.e., first order linear d.e., higher order linear d.e. of constant coefficients. Series: Tests for convergence of numerical series, power series, Taylor series.
Functions of several variables: Limits, continuity. Differentiability, directional derivatives, chain rule. Higher partial derivatives and higher differentials. Extreme value problems. Calculation of double and triple integrals. Transformations of integrals, Jacobi matrix.
Analysis of complex functions: Continuity, regularity, Cauchy - Riemann partial differential equations. Elementary functions of complex variable, computation of their values. Complex contour integral. Cauchy - Goursat basic theorem of integrals and its consequences. Integral representation of regular functions and their higher derivatives (Cauchy integral formulae).

Subject code	Subject name	Requirement	ECTS credit
BMETE90AX59	Mathematics A2 for Electrical Engineers	Exam	6

Course type	Course code	Course language	Timetable information
Lecture	EN1	English	MON:10:15-12:00(IB145); WED:08:15-10:00(IB145)
Practice	EN2	English	WED:10:15-12:00(V1103)

Subject code	Subject name	Requirement	ECTS credit
BMETE91AM43	Informatics 2	Mid-semester mark	4

Course type	Course code	Course language	Timetable information
Laboratory	EN1	English	WED:10:15-12:00(H601)
Lecture	EN0	English	WED:13:15-14:00(H406)

The course aims to learn the programming through understanding the Python language. Introduction to programming and Python language, data types, expressions, input, output. Control structures: if, while. Flowchart, structogram, Jackson figures. Complex control structures. Fundamental algorithms (sum, selection, search extrema, decision..., many practical examples). Lists. For cycle. Newer algorithms (sorting, splitting into two lists...). Exception handling. Abstraction of a part of the program, name it, using as a building block = function. Function call process, parameters, local variables, passing by value. Abstraction: complex data types from simple ones, for example fraction (numerator + denominator), complex numbers (real & imaginary part). OOP concepts: object, method. File management. Command-line arguments. Recursion (painting of an area, building a labyrinth). Algorithms efficiency, quick sorting, binary search versus linear search, O(n). Data structures: binary tree (algorithms), effectiveness: search trees (Morse tree). Mathematical libraries. Modules.

Subject code	Subject name	Requirement	ECTS credit
BMETE91AM44	Informatics 3	Mid-semester mark	4

Course type	Course code	Course language	Timetable information
Laboratory	EN1	English	THU:08:15-10:00(H207)
Lecture	EN0	English	TUE:08:15-10:00(H405A)

The aim of the course is to understand the basic elements of C++ language fundamental in effective scientific calculations. Compiling C++ programs, programming environments for C++. Input/Output. Built-in data types: int, double, char, bool, complex. Control commands: if, switch, for, while, do. Exception handling(recall Python). Functions. Extending operators (fractions struct), references (a += b, cout << fraction, cin >> fractions). Object-oriented programming in C++: object, class, encapsulation, member functions, constructors, destructors (in complex class with re + im or r + fi data members). Using arrays in C++. Pointers, relationship with arrays. File management. Basic algorithms: search, sort, etc. Command-line arguments. Dynamic memory management, new[], delete[]. Inheritance. Templates. Libraries. Header files.
– E. Scheinerman: C++ for Mathematicians. An Introduction for Students and Professionals, CRC Press

Subject code	Subject name	Requirement	ECTS credit
BMETE91AM57	Programming Exercises forTheory of Algorithms	Mid-semester mark	2

Course type	Course code	Course language	Timetable information
Laboratory	TA	English	MON:10:15-11:00(H207)

The aim of the course is to maintain the students' programming skills through programming problems associated with the topics of Algorithm Theory course helping the understanding of the basic concepts of algorithms.
– M. L. Hetland: Python Algorithms, Mastering Basic Algorithms in the Python Language, Apress, 2010.

Subject code	Subject name	Requirement	ECTS credit
BMETE91AM59	Number Theory	Exam	2

Course type	Course code	Course language	Timetable information
Lecture	T0	English	WED:16:15-18:00(H607)

Basic Number Theory: Divisibility, greatest common divisor, Euclid's algorithm, congruences, Chinese remainder theorem, Hensel lifting, primitive roots, discrete logarithm, quadratic residues, Legendre and Jacobi symbol. Law of quadratic reciprocity.
Analytic Number Theory: Prime numbers and its properties, primes of special forms. Primes in arithmetic progressions, gaps between primes, Bertrand's postulate, the Prime Number Theorem. The Riemann zeta function, Riemann Hypothesis, Dirichlet characters. The generating function and its applications, partitions. Sieve methods, application of Brun's sieve to estimate the number of twin primes, Goldbach's conjecture. Additive and multiplicative arithmetic functions. Additive Number Theory: Sumsets, direct and inverse problems. Sum-product estimates.
Combinatorial Number Theory: Schnirelman density, Schur's theorem, van der Waerden's theorem, Szemerédi's theorem about arithmetic progressions. Zero-sum combinatorics: the polynomial method, Combinatorial Nullstellensatz, applications.
Diophantine equations: sum of two, three, four squares, representations as the sums of k-th powers, Waring problem. Fermat's last theorem. Mordell equation. The abc conjecture.
Miscellaneous modern topics (sketch only): Number Theory in Cryptography: The RSA and the ElGamal scheme. Primality tests. Diophantine Approximation Theory: Continued fractions. Pell equation. Wiener attack against RSA. p-adic numbers.

Subject code	Subject name	Requirement	ECTS credit
BMETE93BG02	Mathematics G2	Exam	6

Course type	Course code	Course language	Timetable information
Lecture	EN0	English	TUE:16:15-19:00(KF81); WED:16:15-17:00(KF81)
Practice	EN1	English	WED:17:15-19:00(KF81)

Solving systems of linear equations:elementary row operations, Gauss-Jordan- and Gaussian elimination. Homogeneous systems of linear equations. Arithmetic and rank of matrices. Determinant:geometric interpretation, expansion of determinants. Cramer's rule, interpolation, Vandermonde determinant. Linear space,subspace, generating system, basis, orthogonal and orthonormal basis. Linear maps, linear transformations and their matrices. Kernel, image, dimension theorem. Linear transformationsand systems of linear equations. Eigenvalues, eigenvectors, similarity, diagonalizability. Infinite series:convergence, divergence, absolute convergence. Sewuences and series of functions, convergence criteria, power series, Taylor series. Fourier series:axpansion, odd and even functions. Functions in several variables:continuity, differential and integral calculus, partial derivatives, Young's theorem. Local and global maxima / minima. Vector-vector functions, their derivatives, Jacobi matrix. Integrals: area and volume integrals.

Subject code	Subject name	Requirement	ECTS credit
BMETE94AM22	Convex Geometry	Exam	4

Course type	Course code	Course language	Timetable information
Lecture	E0	English	FRI:10:15-12:00(H601)
Practice	E1	English	FRI:12:15-14:00(H601)

Introduction: affine and convex sets, affine dependence, independence, affine and convex combinations, affine hull, isolation theorem, characterization of closed, convex sets as the intersection of closed half spaces. Convex hull, theorems of Radon, Helly and Carathéodory, their applications. Linear functionals and their connection with hyperplanes, Minkowski sum, separation of convex sets with hyperplanes, supporting hyperplanes, faces of a convex body, extremal and exposed points, theorems of Krein-Milman and Straszewicz. Indicator function, algebras of closed/compact convex sets, valuations, Euler characteristic and the proof of its existence. Convex polytopes and polyhedral sets, their connection, face structure of polytopes, combinatorial equivalence. The f-vector of polytopes, Euler characteristic of polytopes, theorem of Euler. Polar of a set, fundamental properties of polarity, properties of the polar of a polytope, dual polytope. Moment curve, cyclic polytopes and their face structure, Gale’s evenness condition. Hausdorff distance of convex bodies. Affine transformations, Banach-Mazur distance. Ellipsoid as an affine ball. Unique existence of largest volume inscribed, and smallest volume circumscribed ellipsoid of a convex body. The Löwner-John ellipsoid, John’s theorem for general, and centrally symmetric convex bodies.
– Branko Grünbaum: Convex Polytopes, Graduate Texts in Mathematics 221, Springer, New York, 2003.

Subject code	Subject name	Requirement	ECTS credit
BMETE94AM26	Differential Geometry 1	Mid-semester mark	5

Course type	Course code	Course language	Timetable information
Lecture	E0	English	TUE:12:15-14:00(T606)
Practice	E1	English	THU:12:15-14:00(H601)

Curves,reparameterization, length. Tangent line, osculating planes, curves of general type. Frenet frame, Frenet's formulas, curvatures. The fundamental theorem of curve theory. Plane curves: osculating circle, evolute, involutes, parallel curves. Rotation number, Hopf's theorem. Convex curves, the four vertex theorem. Curves in space: osculating, normal and rectifying planes, geometrical interpretation of curvatures.Hypersurfaces,parameterization, tangent plane, normal curvature, Meusnier's theorem. Fundamental forms, Weingarten map. Principal Axis Theorem, principal curvatures, Gaussian and mean curvature. Umbilical points, surfaces of rotation, ruled surfaces. Gauss frame, Christoffel symbols, Gauss and Codazzi–Mainardi equations. The fundamental theorem of hypersurface theory, Theorema Egregium. Tensor fields, Riemannian curvature tensor, Bianchi identity.

Manfredo Do Carmo: Differential Geometry of Curves and Surfaces
Szőkefalvi-Nagy Gyula, Gehér László, Nagy Péter: Differenciálgeometria (1979)Balázs Csikós: Differential GeometryV.T. Vodnyev: Differenciálgeometriai feladatgyűjtemény

Subject code	Subject name	Requirement	ECTS credit
BMETE95AM30	Probability Theory 2	Exam	4

Course type	Course code	Course language	Timetable information
Lecture	T0	English	MON:10:15-12:00(H601)
Practice	T1	English	TUE:12:15-14:00(H405A)

Subject code	Subject name	Requirement	ECTS credit
BMETE95MM07	Markov Processes and Martingales	Exam	5

Course type	Course code	Course language	Timetable information
Lecture	T0	English	TUE:08:15-10:00(H406)
Practice	T1	English	WED:08:15-10:00(H406)

1. Martingales:Review (conditional expectations and tower rule, types of probabilistic convergences and their connections, martingales, stopped martingales, Doob decomposition, quadratic variation, maximal inequalities, martingale convergence theorems, optional stopping theorem, local martingales). Sets of convergence of martingales, the quadratic integrable case. Applications (e.g. Gambler's ruin, urn models, gambling, Wald identities, exponential martingales). Martingale CLT. Azuma-Höffding inequality and applications (e.g. travelling salesman problem)2. Markov chains:Review (definitions, characterization of states, stationary distribution, reversibility, transience-(null-)recurrence). Absorbtion probabilites. Applications of martingales, Markov chain CLT. Markov chains and dynamical systems; ergodic theorems for Markov chains. Random walks and electric networks
3. Renewal processes:Laplace transform, convolution. Renewal processes, renewal equation. Renewal theorems, regenerative processes. Stationary renewal processes, renewal paradox. Examples: Poisson process, applications in queueing4. Point processes:Definition of point processes. The Poisson point process in one and more dimensions. Transformations of the Poisson point process (marking and thinning, transforming by a function, applications). Point processes derived from the Poisson point process. 5. Discrete state Markov processes:Review (infinitesimal generator, connection to Markov chains, Kolmogorov forward and backward equations, characterization of states, transience-(null-)recurrence, stationary distribution). Reversibility, MCMC. Absorption probabilities and hitting times. Applications of martingales (e.g. compensators of jump processes). Markov processes and dynamical systems; ergodic theorems for Markov processes. Markov chains with locally discrete state space: infinitesimal generator on test functions

References:Karlin, S.; Taylor, H. M.: Sztochasztikus folyamatok. Gondolat Kiadó, Budapest, 1985Lindvall, T.: Lectures on the Coupling Method. Dover Publications, Inc., Mineola, NY, 2002Norris, J. R.: Markov chains. Cambridge University Press, Cambridge, 1998 Resnick, S.: Adventures in Stochastic Processes. Birkhäuser Boston, 1992 Rosenblatt, M.: Markov processes. Structure and Asymptotic Behavior. Springer-Verlag, New York-Heidelberg, 1971 Williams, D.: Probability with Martingales. Cambridge University Press, 1991

Subject code	Subject name	Requirement	ECTS credit
BMETE95MM41	Stochastic Analysis	Exam	8

Course type	Course code	Course language	Timetable information
Lecture	T0	English	MON:12:15-14:00(H405A); WED:10:15-12:00(H405A)
Practice	T1	English	WED:14:15-16:00(H405A)

1) Martingales, discrete stochastic integral, optional stopping theorem, discrete Doob decomposition.2) Multivariate normal distribution, Gaussian process, Paul Lévy's construction of Brownian motion.3) Martingales derived from

Brownian motion, properties of Brownian motion, B.M. is nowhere differentiable.4) Stieltjes integral, quadratic variation (e.g. of Brownian motion), mutual variation.5) Strong Markov property, reflection principle for Brownian motion.6) Definition of Ito integral (w.r.t. Brownian motion), case of deterministic integrand (Gaussian process), martingale property of Ito integral, quadratic variation of Ito integral.7) Def of Ito process, Ito formula (in the case when we integrate w.r.t. B.M.)8) Stochastic integral w.r.t. Ito process, Ito formula for Ito processes.9) Stochastic integration by parts, time-dependent Ito formula, multivariate Ito formula.10) Harmonic functions and martingales.11) Paul Lévy's characterization of B.M.12) Martingale representation theorem.13) Existence and uniqueness of strong solution of stochastic differential equation.14) Famous stochastic differential equations (SDEs): O-U process, Geometric Brownian motion, Brownian bridge.15) Equivalent definitions and properties of Besssel process, relation of squared Bessel process and branching processes.16) Stochastic exponential and stochastic logarithm.17) General linear SDE, stochastic logistic equation, CIR process.18) Infinitesimal generator of diffusion process, Dynkin's formula.19) Weak solution of SDE, Tanaka's counterexample, Tanaka's formula.20) Diffusions and related elliptic PDE's(Laplace, Poisson, Helmholtz).21) Diffusions and related parabolic PDE's (heat equation, Kolmogorov forward/backward, Feynman-Kac formula).22) Stationary distribution of 1-dimensional diffusion process.23) Change of measure, Girsanov's formula.
– H-H. Kuo, Introduction to Stochastic Integration, Springer, 2008.– F.C. Klebaner, Introduction to stochastic calculus with applications, (Third edition) Imperial College Press, 2012.– Durrett, Richard. Stochastic calculus: a practical introduction. Vol. 6. CRC press, 1996.

Subject code	Subject name	Requirement	ECTS credit
BMETEAGBsMBAL2-00	Introduction to Algebra 2	Exam	8

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	EN0	English	
Practice	EN1	English	

Scalar product and its properties in R^n. Orthogonal and orthonormal bases, Gram-Schmidt orthogonalization, orthogonal matrices, orthogonal transformations. Householder reflection, Givens rotation. Existence and determination of QR decomposition. Optimal solution of a linear systems of equations using QR decomposition. Scalar product in C^n. Unitary, normal and self-adjoint matrices and transformations. Eigenvalues, eigenvectors and eigenspaces of matrices and transformations. Characteristic polinomial, solution of the eigenvalue problem. Applications. Algebraic and geometric multiplicity, eigenvalues of special matrices, eigenvalues of similar matrices. Cayley-Hamilton theorem. Diagonalizability of matrices and equivalent formulations (real and complex case), diagonalizability of special matrices, relation with eigenvalues, unitary and orthogonal diagonalizability, Schur decomposition, spectral decomposition.
Bilinear functions, standard form, signature, principal axis theorem. Definition of quadratic forms. Classification of local extrema, geometric applications and illustration. Multilinear functions and maps, the total derivative as a multilinear function, multivariable Taylor formula, the determinant as a multilinear function. Singular Value Decomposition, polar decomposition, applications of SVD, pseudoinverse from SVD. Normal forms of matrices, existence, uniqueness and determination, generalized eigenvectors, Jordan chain and Jordan basis. Norms of real and complex vectors, matrix norms, basic properties and determination, functions of matrices (convergence only at the level of mention and illustration), exponential functions of matrices. Vector spaces over arbitrary fields. Existence of basis, dimension, infinite dimensional examples (function spaces, etc.), isomorphism of vector spaces. Concept, properties, isomorphism of Euclidean spaces. Dual space. Applications of vector spaces over finite fields in code theory, cryptography, combinatorics.
Wettl F.: Lineáris algebra. Typotex 2023.
V. V. Praszolov: Lineáris Algebra. Typotex 2004.
Kiss E.: Bevezetés az algebrába. Typotex 2007.

Subject code	Subject name	Requirement	ECTS credit
BMETEAGBsMBALG-00	Introducion to Algebra	Exam	8

Course type	**Course code**	**Course language**	**Timetable information**
Lecture	A0	English	TUE:14:15-16:00(H405A); WED:14:15-16:00(H601)
Practice	A1	English	WED:16:15-18:00(H406); THU:14:15-16:00(H406)

The mathematics of integers: divisibility, division with remainder, greatest common divisor, Euclidean algorithm, irreducible and prime numbers, the fundamental theorem of number theory. Linear Diophantine equations, modular arithmetic, complete and reduced residue systems, solving linear congruences. Fields of prime order. Irreducibility of polynomials and unique factorization. Schönemann-Eisenstein criterion. Multivariate polynomials, complete and elementary symmetric polynomials, relations between roots and coefficients.
Cayley-Hamilton theorem. Bilinear forms, symmetric and symplectic bilinear functions. Standard form, signature, principal axis theorem. Quadratic forms. Classification of local extrema, geometric applications and illustration. Unitary and normal matrices, complex spectral theorem. Polar decomposition, applications of SVD, pseudoinverse and its properties. Normal forms of matrices, existence, uniqueness and computation, generalized eigenvectors, Jordan chain and Jordan basis. Norms of real and complex vectors, matrix norms, basic properties and computation, functions of matrices (convergence only mentioned and illustrated), exponential functions of matrices. Vector spaces over arbitrary fields. Existence of basis, dimension, infinite dimensional examples (function spaces, etc.),

isomorphism of vector spaces. Notion, properties, isomorphism of Euclidean space. Dual space. Applications of vector spaces over a finite field in coding theory, cryptography, combinatorics.
S Roman: Advanced Linear Algebra. Springer 2008.
R. Irving: Integers, Polynomials, and Rings - A Course in Algebra. Springer 2004.

Subject code	Subject name	Requirement	ECTS credit
BMETEAGBsMGE2E-00	Geometry 2e	Exam	4

Course type	Course code	Course language	Timetable information
Lecture	E0	English	MON:10:15-12:00(H306)
Practice	E1	English	MON:12:15-14:00(H507)

Isometries: planar and spatial classification, matrix representation, homogeneous coordinates, classification of similarities; Regular polygons and polyhedra: Euler theorem, Platonic and Archemedean solids, Cauchy rigidnes theorem; Conic sections: Dandelin spheares, excentricity, classification by quadratic forms; Introduction to projective geometry: axioms, Desargues theorem, Pappus-Pascal-theorem, perspectivity and projectivity; Classical Eucledean theorems from higher geometry: Ceva and Menelaus theorems.
G.Horváth, Á: Wonderful Geometry, Typotex 2015.
Coxeter, H.S.M: Non-Euclidean Geometry, The Univ. of Toronto Press, 1947.
Smith J. T.: Methods of Geometry , Wiley and Sons, Inc. 2000

Subject code	Subject name	Requirement	ECTS credit
BMETEAGMsMATHA-00	Algebraic Topology and Homological Algebra	Mid-semester mark	5

Course type	Course code	Course language	Timetable information
Lecture	T0	English	THU:14:15-16:00(H507)
Practice	T1	English	TUE:14:15-16:00(H306)

Algebraic topology:1. Simplicial and singular homology2. Basic homological algebra (chains and homotopies)3. Degree, CW-homology4. Cohomology, ring structure5. Orientability, Poincare duality6. Fiber bundles, principal bundles7. Classification of vector bundles, characteristic classes
Homological algebra:1. Categories, functors and natural transformations2. Epimorphism, monomorphism, product and coproduct, additive and abelian categories3. Projective and injective modules4. Exact sequences of modules, exact functors, Snake lemma, 5 lemma5. Hom and Tensor, adjoint properties6. Projective and injective resolutions, Ext and Tor as derived functors of Hom and Tensor7. Ext as a group, Ext and extensions, Hilbert's syzygy theorem
Allen Hatcher, Algebraic Topology, Cambridge University Press
Charles Weibel, An Introduction to Homological Algebra, Cambridge University Press

Subject code	Subject name	Requirement	ECTS credit
BMETEAGMsMDTOP-00	Differential Topology	Exam	5

Course type	Course code	Course language	Timetable information
Lecture	E0	English	THU:12:15-14:00(H507)
Practice	E1	English	THU:10:15-12:00(H406)

Differentiable manifold, orientability, partition of unityTangent bundle, vector field, integral curve, Lie-derivativeRegular and critical values, Sard's lemma, transversalityVector bundles, natural algebraic constructions: direct sum, tensor product, dual, homomorphisms, tensor bundlesDifferential forms, pull-back, exterior product, exterior derivativeIntegration on smooth oriented compact manifolds, Stokes' theoremde Rham cohomology, cohomology with compact supports, functoriality propertiesPoincaré's lemmaMayer-Vietoris exact sequencePoincaré duality, degree of a mapKünneth's formulaChern classes
Bott, Tu: Differential Forms in Algebraic Topology
Dubrovin, Fomenko, Novikov: Modern Geometry
Berger, Gostiaux: Géometrie Differentielle
Gehér, Nagy, Szőkefalvi-Nagy: Differenciálgeometria
Hirsch: Differential Topology
Milnor: Topology from a differential viewpoint

Subject code	Subject name	Requirement	ECTS credit
BMETESZMsMSMOD-00	Stochastic Models	Exam	5

Course type	Course code	Course language	Timetable information
Laboratory	T1	English	
Lecture	T0	English	

– Pólya’s theorem on recurrence versus transience of simple random walk on Z^d.

Green's function. The spectral radius of the d-regular tree.– Fekete's subadditive lemma, with three applications: return probabilities; the connective constant and the speed of random walks on infinite transitive graphs.– Chernov's and Azuma-Hoeffding large deviations inequalities. Comparison with the Strong Law of Large – Numbers and the Central Limit Theorem.– Stochastic domination and couplings.– Erdős-Rényi random graph phase transitions: subgraph containment, connectivity, giant cluster. First and second moment methods. Critical Galton-Watson trees die out: integer-valued Chung-Fuchs theorem for the recurrence of the exploration random walk.– Basics of network science: clustering coefficient, isoperimetric ratio (or Cheeger constant), centrality measures: eigenvector centrality, PageRank.– The basics of Markov chain mixing times: spectral and coupling methods.– The Barabási-Albert preferential attachment graph, and its degree distribution.– Renewal processes: SLLN, renewal equations, renewal theorems, renewal paradox, size-biasing.– Blow-up vs null-recurrence vs positive recurrence in a G/G/1 queuing system.– Copulas of multivariate continuous distributions.– Percolation theory: definitions and their equivalence. Examples: p_c(p_c(p_c([1/3,2/3] using the Peierls contour method.– The Ising model on finite graphs: definition, spatial Markov property, basic properties of the partition function, definition of long range order. The Curie-Weiss phase transition.

– R. Durrett: Essentials of stochastic processes, 2nd edition. Springer, 2011. https://services.math.duke.edu/~rtd/EOSP/EOSP2E.pdf
– R. Durrett: Random graph dynamics. Cambridge University Press, 2007. https://www.math.duke.edu/~rtd/RGD/RGD.pdf.
– R. Durrett. Probability: theory and examples. 5th edition. Cambridge University Press, 2019. https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf
– D. Levin, Y. Peres, E. Wilmer: Markov chains and mixing times. American Mathematical Society, 2008. http://pages.uoregon.edu/dlevin/MARKOV/.
– M. Newman: Networks. An introduction. Oxford University Press, 2010.
– G. Pete: Probability and geometry on groups, http://math.bme.hu/~gabor/PGG.pdf